

Maximizing the Profit of a Business

Math 1010 Intermediate Algebra Group Project

Background Information:

Linear Programming is a technique used for optimization of a real-world situation. Examples of optimization include maximizing the number of items that can be manufactured or minimizing the cost of production. The equation that represents the quantity to be optimized is called the objective function, since the objective of the process is to optimize the value. In this project the objective is to maximize the profit for a company that manufactures furniture.

The objective is subject to limitations or constraints that are represented by inequalities. Limitations on the number of items that can be produced, the number of hours that workers are available, and the amount of money a company has for advertising are examples of constraints that can be represented using inequalities. Making and selling an infinite amount of furniture is not a realistic goal. In this project the constraints will be based on the number of weekly work hours available in two departments that construct the furniture.

Graphing the system of inequalities based on the constraints provides a visual representation of the amount of furniture that it is feasible to make in a week. If the graph is a closed region, it can be shown that the values that optimize the objective function will occur at one of the "corners" of the region.

The Problem:

In this project your group will solve the following situation:

A manufacturer produces the following two items: computer desks and bookcases. Each item requires processing in each of two departments. In the cutting department the pieces of wood required for each item are cut out. The assembly department is where the pieces are assembled into desks and bookcases. The cutting department has 48 hours available and the assembly department has 36 hours available each week for production. To manufacture a computer desk requires 4 hours of cutting and 4 hours of assembly while a bookcase requires 3 hours of cutting and 2 hours of assembly. Profits on the items are \$56 and \$38 respectively. If all the units can be sold, how many of each should be made to maximize profits?

Modeling the Problem:

Let X be the number of computer desks that are sold and Y be number of bookcases sold.

1. Write down a linear inequality for the hours used in cutting.

2 $4x + 3y < 48$

2. Write down a linear inequality for the hours used in assembly.

2 $4x + 2y < 36$

3. There are two more constraints that must be met. These relate to the fact that the manufacturer cannot produce negative numbers of items. Write the two inequalities that model these constraints:

$$x \geq 0$$

$$y \geq 0$$

4. Next, write down the profit function for the sale of X desks and Y bookcases. This is the Objective Function for the problem.

5

$$P = 56x + 38y$$

You now have four linear inequalities and a profit function. These together describe the manufacturing situation. These together make up what is known mathematically as a **linear programming problem**. Write all of the inequalities and the profit function together below. This is typically written as a list of constraints, with the profit function last.

last. $4x + 3y < 48$

$$4x + 2y < 36$$

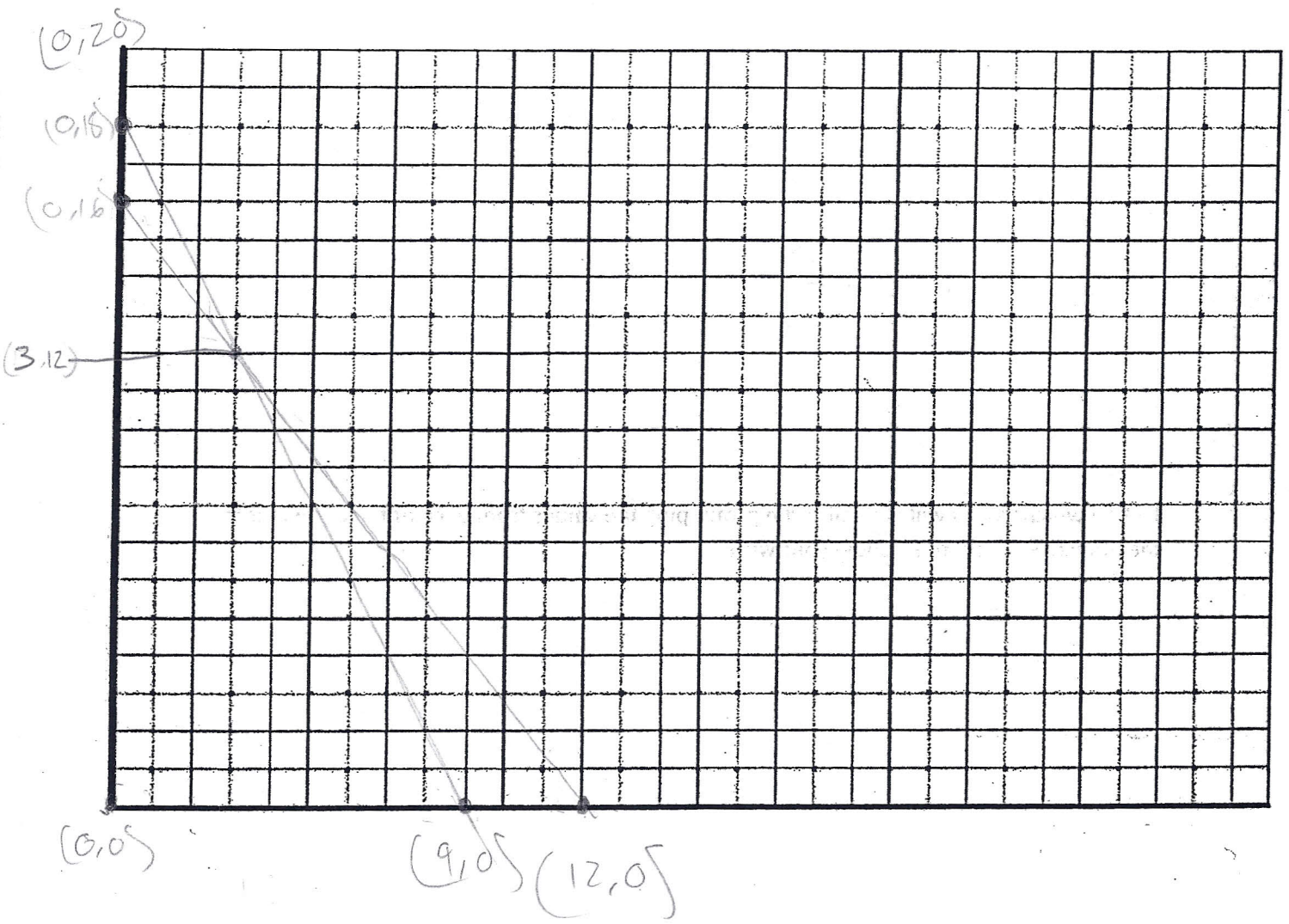
$$x \geq 0$$

$$y \geq 0$$

$$P = 56x + 38y$$

5. To solve this problem, you will need to graph the **intersection** of all four inequalities on one common XY plane. Do this on the grid below. Have the bottom left be the origin, with the horizontal axis representing X and the vertical axis representing Y.

3.5



- 4 6. The above graph is called the feasible region. Any (x, y) point in the region corresponds to a possible number of computer desks and bookcases that can be manufactured in a week. However, the values that will maximize the profit occur at one of the vertices or corners of the region. Your region should have four corners. Find the coordinates of the ordered pairs of these corners. Be sure to show your work and label the (x, y) coordinates of the corners in your graph.

$$\begin{array}{l}
 (0, 16) \quad (0, 18) \quad (0, 0) \\
 \times (12, 0) \quad (9, 0) \quad (3, 12)
 \end{array}$$

$$\begin{array}{r}
 4x + 3y = 48 \\
 -4x \qquad -4x \\
 \hline
 3y = 48 - 4x \\
 \frac{3y}{3} = \frac{48 - 4x}{3} \\
 y = \frac{48 - 4x}{3}
 \end{array}$$

$$\begin{array}{r}
 4x + 2y = 36 \\
 -4x \qquad -4x \\
 \hline
 2y = 36 - 4x \\
 \frac{2y}{2} = \frac{36 - 4x}{2} \\
 y = \frac{36 - 4x}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{LCD} = 6 \quad (6) \frac{48 - 4x}{3} = \frac{36 - 4x}{2} \quad (6) \\
 2(48 - 4x) = 3(36 - 4x) \\
 96 - 8x = 108 - 12x \\
 -96 \quad -96 \quad -96 \\
 -8x = 12 - 12x \\
 +12x \quad +12x \\
 \hline
 4x = 12 \\
 \frac{4x}{4} = \frac{12}{4} \quad x = 3
 \end{array}$$

$$\begin{array}{r}
 y = \frac{48 - 4(3)}{3} \\
 y = \frac{48 - 12}{3} \\
 y = \frac{36}{3} \\
 y = 12
 \end{array}$$

- 4 7. To find which values will maximize the profit, plug the values from each of the corners into the objective function, P . Show your work.

$$\begin{array}{ll}
 P = 0 + 16(38) & P = 608 \\
 \cancel{P = 0 + 18(38)} & \cancel{P = 684} \\
 \cancel{P = 12(56) + 0} & \cancel{P = 672} \\
 P = 9(56) + 0 & P = 504 \\
 P = 36(0) + 39(0) & P = 0 \\
 P = 36(3) + 39(12) & P = 624
 \end{array}$$

8. Write a sentence describing how many of each type of furniture you should build and sell and what is the maximum profit you will make.

4 $P = 56^{(3)} + 38^{(12)}$

$$P = 168 + 456$$

I should build and sell three computer desks & twelve Book cases to maximize my profits.

9. Reflective Writing.

6 Did this project change the way you think about how math can be applied to the real world? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.

This project changed the way I think about how math can be applied to the real world by showing that math is not just subject that we have to learn but can actually be used to help us solve everyday problems of our lives.